MATH 121
Functions—Function's Definition

1. (Section 1.2) For each part give an example of a function, if one exists, with the set of inputs (domain) = *A* and the possible outputs coming from *B*, satisfying the stated condition. (Note: For each function, your definition must say what the function does to each input. i.e. For each input what is the output?) Note: $\Re$ is the set of real numbers.
   a) $A = \{1,2,3\}$, $B = \{\sqrt{7},1,0,4\}$ and *f* has exactly 2 outputs.
   b) $A = \{\text{red, blue}\}$, $B = \Re$, the real numbers, and (i) *f* has exactly 1 output. (ii) *f* has exactly two outputs. (iii) *f* has 3 outputs.
   c) $A = \Re$, $B = \{0,1\}$ and *f* has exactly 2 outputs. (Give 2 examples.)
   d) $A = \{1,2,3\}$, $B = \{\text{a, b, c, } d\}$ and no two inputs have the same output. (Give two examples) (Remark: Such a function is called "one – to – one.")
   e) $A = \Re$, $B = \{x | x$ is a real number and $x \geq 0\}$ and every number in B, "gets hit" by some number in A. (Remark: Such a function is called "onto".)

2. (Section 2.4) Consider a function *f* with the following properties
   i) Domain of *f* is all real numbers, $\Re$.
   ii) Range of *f* is the interval $[-1,1]$.
   iii) $f(-x) = f(x)$ for all inputs *x* (*f* is called an "even" function)
   a) Describe *f* with a formula (or formulas)
   b) Describe *f* verbally. (The function should be different than in part (a)
   c) Describe *f* with a table (different function again.)

3. (Section 1.2) Being able to recognize patterns is an invaluable problem-solving technique, and it helps immensely in the study of the concept of a function. In table below find a formula for $f(x)$.

x	1	2	3	4	-1	-2	-3
$f(x)$	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{7}{4}$	2	undefined	8	$\frac{11}{2}$

4. (Section 1.2) Sketch the graph of a function *f* having the given domain, *D*, and range *R*.
   a) $D = \Re$; $R = (-3,3)$
   b) $D = (-4,4)$; $R = [-3,5]$
   c) $D = [-2, 3)$; $R = \Re$
   d) $D = \Re$; $R = \{-1,1\}$

5. (Section 2.4) Find the function *f* such that $f(x+3) = f(x)$ for all numbers *x*.

6. (Section 2.4) A piecewise-defined function *f* has a domain of $[0,6]$ and a range of $(2,5)$. Give a possible formula for this function *f*. (Hint: Draw a picture…be creative.)

7. (Section 2.4) Give a sketch of the graph of a <u>single</u> function f which meets <u>ALL</u> of the following conditions:

(a) The domain of f is $[-3,8]$

(b) The range of f is $(-1,6)$

(c) $f(-2)=f(1)$

(d) $f(1)=\dfrac{1}{f(-3)}$

(e) $f(5)=2f(-1)$

8. (Section 2.4) Give a sketch of a graph of <u>**ONE**</u> function meeting <u>**ALL**</u> of the following conditions:

a) The domain of f is $(-\infty,-1]\cup(4,+\infty)$

b) The range of f is $[0,+\infty)$

c) $f(-3)\cdot f(6)=10$

d) $f(-3)+f(-1)=1$

e) $f(-4)=2+f(7)$

f) $f(-2)=\sqrt{f(-4)}$

9. (Section 4.4) For each of the following, give an example of a polynomial function that satisfies the given condition(s):

a) f has degree 2 and $f(0)=2$

b) f has degree 3, $f(0)=2$ and $f(1)=4$

c) f has degree 1, $f(0)=2$ and $f(1)=5$

10. (Section 4.4) Give an example of a cubic polynomial function *f*, if one exists, whose graph has:

a. $3\ x-$intercepts
b. $2\ x-$intercepts
c. $1\ x-$intercepts

d. 0 x – intercepts

11. (Section 4.4) Give an example of a 4^{th} – degree polynomial function P (with real coefficients), if one exists, which has:
a) Exactly 4 distinct real roots, and each root of multiplicity 1.
b) Exactly 3 distinct real roots, and each root multiplicity 1.
c) Exactly 2 distinct real roots, and each root of multiplicity 1.
d) Exactly 1 distinct real root, and this root is of multiplicity 1.
e) Exactly 2 complex roots.

12. (Section 4.4) Give an example of a polynomial function f such that
a) $f(x) > 0$ if $|x| > 4$ or $|x| < 1$, or if $-4 < x < -1$ or $1 < x < 4$.
b) $f(x) > 0$ if $x < -4$ or $-1 < x < 1$ or $x > 1$, $f(x) < 0$ if $-4 < x < -1$ and $F(x) = 0$ otherwise.

13. (Section 4.5) Define a rational function f such that
i) $x = 1$ is a vertical asymptote
ii) $y = 5$ is a horizontal asymptote
iii) y – intercept is 5
iv) There are no x – intercepts

14. (Section 4.5) Give two examples of a rational function with $y = 1$ as an asymptote and passes through the point $(2,\ 2)$.

Answer Keys:

1. Answers vary.
a) $\{(1,1),(2,0),(3,0)\}$
b) (i) $\{(red,2),(blue,2)\}$ (ii) $\{(red,1),(blue,0)\}$ (iii) not possible
c) $f(x) = \begin{cases} 0 & x \geq 0 \\ 1 & x < 0 \end{cases}$
d) $\{(1,a),(2,b),(3,d)\}$
e) $f(x) = x^2$

2. Answers vary.
a) $f(x) = \begin{cases} 1 & x \geq \sqrt{2} \\ x^2 - 1, & -\sqrt{2} < x < \sqrt{2} \\ 1 & x \leq \sqrt{2} \end{cases}$

3. $f(x) = \dfrac{3x-2}{x+1}$

4. Answers vary.

5. Answers vary, f(x) = 4. ☺
6. Answers very. Have fun with this one.
7 and 8. Answers vary, but make sure to give one function that satisfies all conditions.
9. (a) $f(x) = x^2 + 3x + 2$
(b) $f(x) = x^3 + 2x^2 - x + 2$
(c) $f(x) = 3x + 2$
10. (a) $f(x) = (x-3)(x+9)(x+2)$
(b) not possible
(c) $f(x) = (x-3)(x^2+1)$
(d) not possible
11. Answers vary. (a) f(x) = (x – 3)(x – 5)(x + 8)(x + 5)
(b) not possible
(c) f(x) = (x – 4)(x + 3)(x – 3i)(x + 3i)
(d) not possible
(e) $f(x) = (x-2i)^2(x+2i)^2$
12. Answers vary.
13. Answers vary. One possible answer is $f(x) = \dfrac{5x^2+5}{x^2-2x+1}$.
14. Answers vary. One possible answer is $f(x) = \dfrac{3x+4}{3x-1}$.