MATH 121

Review Sheet

The numbers (italics) indicate the chapter in Stewart/Redlin/Watson Precalculus 7th edition where more problems of the same type can be found.

C.1 1. Simplify completely: (a) $\left(\dfrac{2^{-1}+3^{-1}}{4^{-1}}\right)\left(2^{-1}+3^{-1}\right)$ (b) $\sqrt{\left(3\sqrt{2}+\sqrt{3}\right)^2+\left(3-\sqrt{6}\right)^2}$

C.1 2. Simplify completely: (a) $\left(\dfrac{4^{\frac{-1}{2}}-4^{\frac{-3}{2}}}{4^{\frac{-1}{2}}+4^{\frac{-3}{2}}}\right)^2$ (b) $\sqrt{\dfrac{4^{\frac{3}{2}}-4^{\frac{-3}{2}}}{4^{\frac{-3}{2}}}}$ (c) $\left(\dfrac{a^{-2}b^{\frac{3}{2}}}{a^{\frac{-2}{3}}b^{-4}}\right)^{\frac{-3}{4}}$

C.1 3. Rationalize the numerator: $\dfrac{\sqrt{x^2+1}-x}{3}$

C.1 4. Simplify completely: $\dfrac{12x}{2x+1}+\dfrac{3}{-2x^2-x}+\dfrac{5}{x}$

C.1 5. Simplify completely: (a) $\dfrac{2(x-2)^{-1}-2(a-2)^{-1}}{x-a}$ (b) $\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}$

C.1 6. Simplify completely: $\dfrac{8x^2-10x-3}{x^2-9}\div\dfrac{4x^2-12x+9}{3x^2-9x}\cdot\dfrac{6x^2+13x+6}{12x^2+11x+2}$

C.1 7. Simplify completely: (a) $\dfrac{2+\sqrt{-8}}{1+\sqrt{-2}}$ (b) $\left(\sqrt{3}-\sqrt{-4}\right)\left(\sqrt{6}-\sqrt{-8}\right)$

C.1 8. Solve algebraically: (a) $x^{1/2}-4x^{1/4}=-3$ (b) $10x^4-17x^2+3=0$

(c) $\sqrt{3x-5}+1=\sqrt{x+6}$ (d) $x^2=3(x-1)$ (e) $|3x-4|=0.01$

C.2 9. Let $f(x)=6(2x+3)-3(x-5)$. Find all numbers x such that $f(x)=7$.

C.2 10. Let $g(x)=-\dfrac{x+1}{x+3}$ and $f(x)=\dfrac{x}{2x+7}$. Find all numbers x such that $f(x)+g(x)=1$.

C.1 11. Solve for w: $S=2(lw+hw+hl)$

C.1 12. Solve for i: $A=P\left(1+\dfrac{i}{100}\right)^2$

C.1 13. Solve the inequality. Express the solution using interval notation. (a) $-2|x+1|-1<-11$

(b) $\dfrac{3}{x-1}-\dfrac{4}{x}\geq 1$ (c) $\dfrac{(x+3)^2(x-2)(5-x)}{(x+4)(3x+7)}\geq 0$ (d) $x(x-1)<(x+3)(x-2)$

*C.1* 14. Write the equation of a circle if $(1,3)$ and $(-5,7)$ are end points of a diameter.

C.1 15. A car is traveling on a curve that forms a circular arc. The force F needed to keep the car from skidding is jointly proportional to the weight w of the car and the square of its speed s and is inversely proportional to the radius r of the curve. (a) Write an equation that expresses this variation. (b) A car weighing 1600 lb travels around a curve at 60 mi/hr. The next car to round this curve weighs 2500 lb and requires the same force as the first car to keep from skidding. How fast is the second car traveling?

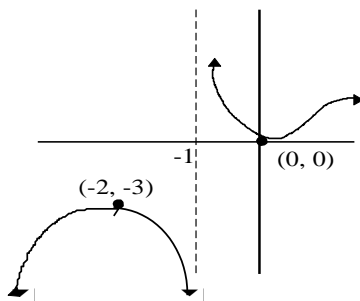*C.4* 16. Find domain of the function given by $f(x)=\dfrac{\ln(x-1)}{\sqrt{16-x^2}}$.

C.2 17. Find $\dfrac{1}{f(2)}$, $f(a+1)$ and the difference quotient $\dfrac{f(x+h)-f(x)}{h}$, where $h\neq 0$.

(a) $f(x)=-x^2-2x+3$ (b) $f(x)=\dfrac{1}{x+1}$ (c) $f(x)=7$ (d) $f(x)=\sqrt{x}$

C.2 18. Given the graph of $y=f(x)$, estimate: (a) $f(x)$ when $x=1$; (b) x when $f(x)=1$.

$y=f(x)$
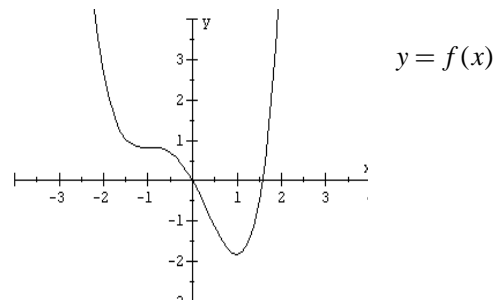
C.2 19. Use the graph to find the domain and range of the function:

Domain _____

Range _____

C.2 20. Find the domain of: $f(x) = \dfrac{\sqrt{2x-5}}{x-7}$. Write your answer using interval notation.

C.2 21. Draw the graph of $f(x)$: $f(x) = \begin{cases} -2x-5 & x < -2 \\ x^2 & -2 \le x \le 3 \\ 3 & x > 3 \end{cases}$

C.2 22. Write the equation of the perpendicular bisector of segment $\overline{AB}$ given $A(3,5)$ and $B(7,6)$.

C.2 23. State the transformations necessary to graph $f(x) = 3(x-2)^2 + 1$ from $f(x) = x^2$, then draw the graph.

C.3 24. Let $f(x) = 2x^2 - 7x + 3$

   (a) Complete the square to find the vertex.

   (b) Find the line of symmetry.

   (c) Find the range of the function
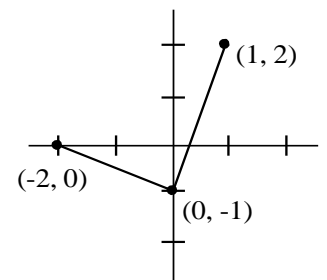
   (d) Does the function have a maximum or minimum?

C.3 25. If a quadratic function has vertex (2, 3) and passes through point (4, 7), find the equation of the function.

C.2 26. If $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 2$, find:

   (a) $(f+g)(x)$ (b) $(fg)(5)$ (c) $(f \circ g)(5)$ (d) $(f \circ g)(x)$

   (e) Find the domain of $(f \circ g)(x)$ and $(g \circ f)(x)$.

C.2 27. Suppose that $f(x)$ is a linear function with $f(3) = -7$ and $f(-5) = -2$. Find $f(x)$.

C.2 28. Given the graph of $y = f(x)$ draw the graph of $y = 2 + 3f(x-1)$.

(1, 2)

(-2, 0)

(0, -1)

C.2 29. If $f(x) = x^2$ write the equation of the quadratic if the following geometric transformations are applied: Vertical stretch by a factor of 2, reflect through the x-axis, horizontal shift 3 left and vertical shift 5 down.

C.2 30. If $g(x) = x^2 + 4x$ with $x \geq -2$, find $g^{-1}(5)$.

C.2 31. Find the rule for the inverse of f if $f(x) = \dfrac{2x-3}{x+5}$.

C.2 32. Draw the graph of f and its inverse on the same set of axes: $f(x) = x^2 + 5$, $x \geq 0$.

C.3 33. Let $f(x) = 2x^3 - 7x^2 - 5x + 4$. (a) Draw a complete graph of $f(x)$. (b) State coordinates of all local maximum and minimum point. (c) State intervals where graph is increasing and decreasing.

C.2 34. State whether the function $f(x) = \dfrac{3x}{x^2 + 5}$ is odd, even, or neither.

C.3 35. Use synthetic division to divide $4x^3 - 7x^2 + 2$ by $x - 3$ and state the quotient and the remainder.

*C.3* 36. Find the remainder if $f(x) = 3x^{75} - 4x + 6$ is divided by $x + 1$.

C.3 37. Find the quotient $Q(x)$ and remainder $R(x)$ if $P(x) = x^3 + 2x^2 + 5$ is divided by $d(x) = x^2 - 4$.

C.3 38. Determine if $x - 3$ is a factor of $f(x) = 2x^3 - 13x^2 + 23x - 6$.

C.3 39. Given that $f(x) = x^2 + (k-1)x - 3k - 17$ has $x + 2$ as a factor, find k.

C.3 40. Write a polynomial of degree 4 with real coefficients and zeros of $-2, 3$, and $4 + i$. Express your result as a product of linear and irreducible quadratic factors with real coefficients.

C.3 41. Write a 2nd degree polynomial f with real coefficients if the graph of $y = f(x)$ is as given:
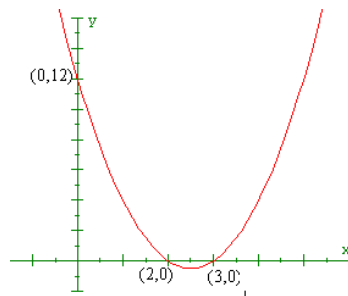
C.2 42. Find the domain and range for the inverse of f if $f(x) = \sqrt{x-3}$.

C.2 43. Let $f(x) = \dfrac{3x-1}{x-2} + \dfrac{4x+5}{x+3}$ and $g(x) = \dfrac{35}{x^2 + x - 6}$.

Find all numbers x such that $f(x) = g(x)$.

C.3 44. Find all zeros of $f(x) = x^4 + 6x^3 - 3x^2 + 24x - 28$ over the field of complex numbers. Write $f(x)$ in the factored form.

C.3 45. Find to the nearest 0.01 all zeros of $f(x) = 3x^3 - 2x^2 - 7x + 3$ over the field of real numbers.

C.3 46. Given the rational function: $f(x) = \dfrac{3x}{x^3 + 5x^2 + 4x}$. (a) List all vertical asymptotes. (b) List all horizontal asymptotes. (c) Find domain of $f(x)$.

C.3 47. Draw a complete graph of $f(x) = \dfrac{x^3 + 3x^2 - 6x - 8}{x^2 - 3x - 10}$. Show all zeros, y-intercepts, vertical asymptotes, horizontal asymptotes, and oblique asymptotes.

C.3 48. Given the graph of $y = f(x)$, write a polynomial function f of least degree:

$y = f(x)$

detail of $y = f(x)$ on $[2.5, 4.5]$
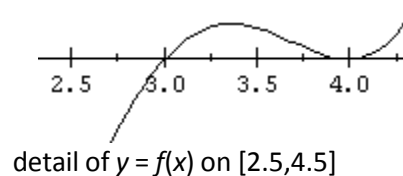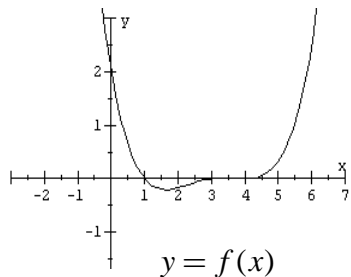
C.2 49. Draw the graph of $f(x) = 3\sqrt{x - 5} - 4$ by applying geometric transformations to the graph of $y = \sqrt{x}$.

C.4 50. Sketch a graph $y = 3^{x+1} - 2$ by applying geometric transformations to the graph of $y = 3^x$. State the domain, range and any asymptotes.

C.4 51. Solve $9^{x^2} = 3^{3x+2}$ without calculator.

C.4 52. Evaluate: (a) $\log_4(-2)$ (b) $\log_3(81)$ (c) $2^{\log_2 8}$

C.4 53. Sketch the graph of $f(x) = \log_5(x+1) - 2$. State the domain, range and asymptotes.

C.4 54. Solve without calculator: $\log_3(7x+3) = \log_3(x-4) + 2$

C.4 55. Express in terms of sums and differences of logarithms: $\log \dfrac{ab^2}{\sqrt{2c}}$

C.4 56. Write the expression as one logarithm: $5\log_4 x - \dfrac{1}{2}\log_4(3x-4) - 3\log_4(5x+1)$

C.4 57. Solve exactly: $5\left(4^{2x-1}\right)=21$

C.4 58. Solve graphically: (a) $4^{2x+3}=5^{x-2}$ (b) $\ln x+\ln(x-2)=5$

C.4 59. Suppose it is given that $\log_3(2)=0.63$ and $\log_3(34)=3.21$. Use these and appropriate properties of logarithms to find the values of the following logarithms.

(a) $\log_3(36)$ (b) $\log_3(32)$ (c) $\log_3\left(\dfrac{1}{17}\right)$ (d) $\log_3\left(\sqrt[3]{68}\right)$

C.4 60. Find the domain of the function: (a) $f(x)=\dfrac{x}{\log x}$ (b) $f(x)=\log_7\left(2-\dfrac{x+5}{x-6}\right)$.

C.3 61. An open box is made from a 10-cm by 20-cm piece of aluminum by cutting a square from each corner and folding up the edges. The area of the resulting base is 90 cm^2. What is the length of the sides of the squares?

C.2 62. A small business purchases a piece of equipment for $25,000. After 10 years, it will have to be replaced. Its value then is expected to be $2,000. Use a linear equation to find when its value will be $13,000.

C.4 63. A certain radioactive substance decays according to a formula $q(t)=q_0e^{-0.0063t}$, where q_0 is the initial amount of the substance and t is the time in days. Approximate the half-life of the substance.

C.4 64. The amount of bacteria in a certain culture triples every 2 hours. Assuming growth is exponential and there are 500 bacteria to start, how many will be present after 6 hours?

C.3 65. A farmer wishes to put a fence around a rectangular field and then divide the field into three rectangular plots by placing two fences parallel to one of the sides. If the farmer can afford only 2000 yards of fencing, what dimensions will give the maximum rectangular area?

C.4 66. A rectangular plot of ground having dimensions 30 feet by 36 feet is surrounded by a walk of uniform width. If the area of the walk is 1000 sq. ft., what is its width?

C.3 67. If a ball is thrown up from the roof of a 100 ft building at an initial velocity of 25ft/sec.

    (a) What is its maximum height? ($s(t)=-16t^2+v_0t+s_0$)

    (b) How long does it take to hit the ground? (round all answers to the nearest hundredth)

C.3 68. The braking distance d (in feet) of a certain car traveling v (mi/hr) is given by $d=v+\dfrac{v^2}{20}$. Determine the velocities that result in braking distance of less than 80 feet.

C.4 69. In May 2010, a 1932 painting, by Pablo Picasso, sold at a New York City art auction for $106.5 million to an anonymous buyer. This is a record price for any work of art sold at auction. The painting had belonged to the estate of Sydney and Francis Brody, who bought it for $17,000 in 1952 from a New York art dealer, who had acquired it from Picasso in 1936. Assuming that the value A_0 of the painting has grown exponentially:

(a) Find the value of k, and determine the exponential growth function $A(t) = A_0 e^{kt}$ assuming $A_0 = 17,000$ and t is the number of years since 1952.

(b) Estimate the value of the painting in 2020.

(c) What is the doubling time for the value of the painting?

(d) In what year will the value of the painting be $240 million, assuming there is no change in the growth rate?

C.3 70. An aquarium of height 2 feet is to have a volume of 10 ft^3. Let x denote the length of the base and y the width.
  a) Express y as a function of x.
  b) Express the total number S of square feet of glass needed as a function of x (4 sides & bottom are glass -- no top).

C.3 71. From a rectangular piece of cardboard having dimensions 25 inches by 33 inches, an open box is to be made by removing squares of area x^2 from each corner and turning up the sides. Find the value of x so the volume is (a) 1400 cubic inches   (b) maximal.

C.2 72. In a certain state the maximum speed permitted on freeways is 65 mi/h, and the minimum is 40 mi/hr. The fine F for violating these limits is $15 for every mile above the maximum or below the minimum.

(a) Complete the expression on the following piecewise defined function, where x is the speed at which you are driving.

$$F(x) = \begin{cases} \rule{3cm}{0pt} & \text{if } 0 < x < 40 \\ \rule{3cm}{0pt} & \text{if } 40 \le x \le 65 \\ \rule{3cm}{0pt} & \text{if } x > 65 \end{cases}$$

(b) Find $F(30)$, F(50), and F(75).

(c) What do your answers in part (b) represent?