MATH 121

Review Sheet

The numbers (italics) indicate the section in Bittinger, Beecher, Ellenbogen, Penna 5th edition where more problems of the same type can be found.

R.2 1. Simplify completely: $\left(\frac{2^{-1}+3^{-1}}{4^{-1}}\right)\left(2^{-1}+3^{-1}\right)$

R.7 2. Simplify completely: (a) $\left(\frac{4^{-\frac{1}{2}}-4^{-\frac{3}{2}}}{4^{-\frac{1}{2}}+4^{-\frac{3}{2}}}\right)^2$ (b) $\sqrt{\frac{4^{\frac{3}{2}}-4^{-\frac{3}{2}}}{4^{-\frac{3}{2}}}}$ (c) $\left(\frac{a^{-2}b^{\frac{3}{2}}}{a^{-\frac{2}{3}}b^{-4}}\right)^{-\frac{3}{4}}$

R.7 3. Simplify completely: $\sqrt{a}\,\sqrt[3]{a^2}$

R.7 4. Simplify completely: $\sqrt{\left(3\sqrt{2}+\sqrt{3}\right)^2+\left(3-\sqrt{6}\right)^2}$

R.7 5. Rationalize the denominator and simplify a) $\sqrt[3]{\frac{16x^5}{y}}$ b) $\frac{1-\sqrt{x}}{2+\sqrt{x}}$

R.6 6. Simplify completely: $\frac{2(x-2)^{-1}-2(a-2)^{-1}}{x-a}$

R.6 7. Simplify completely: $\frac{8x^2-10x-3}{x^2-9} \div \frac{4x^2-12x+9}{3x^2-9x} \bullet \frac{6x^2+13x+6}{12x^2+11x+2}$

R.6 8. Simplify completely: $\frac{12x}{2x+1}-\frac{3}{2x^2+x}+\frac{5}{x}$

R.6 9. Simplify completely: $\frac{\frac{5}{x+1}+\frac{2x}{x+3}}{\frac{7}{x+3}+\frac{x}{x+1}}$

1.2 10. Let $f(x)=6(2x+3)-3(x-5)$. Find all numbers x such that $f(x)=7$.

3.4 11. Let $g(x)=\frac{5}{x-4}$ and $f(x)=\frac{3}{2x+1}$. Find all numbers x such that $f(x)=g(x)$.

R.5 12. Solve for w: $S=2(lw+hw+hl)$

R.5 13. Solve by factoring: $6x^2 - x = 15$

3.2 14. Solve by quadratic formula and give exact answer: $5x^2 - 7x - 3 = 0$

3.2 15. Solve by graphing and give answers to nearest .01: $3x^2 - 5x - 4 = 0$

3.2 16. Solve algebraically: (a) $x^{1/2} - 4x^{1/4} = -3$ (b) $10x^4 - 17x^2 + 3 = 0$

1.6 17. Solve and give answer in interval form: $x(x-1) < (x+3)(x-2)$

1.6 18. Solve and give answer in interval form: $-1 < \dfrac{7 - 2x}{5} \le 4$

3.5 19. Let $f(x) = -2|x+1| - 1$. Find all numbers x, if there are any, such that $f(x) < -11$. Write answer in interval form.

4.6 20. Solve and give answer in interval form: $\dfrac{(x+3)^2(x-2)(5-x)}{(x+4)(3x+7)} \ge 0$

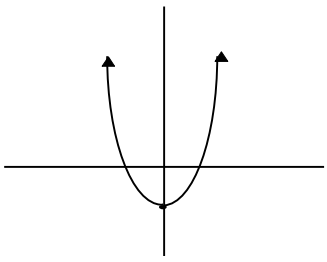
4.6 21. Solve and give answer in interval form: $3x^3 - 8x^2 - 9x + 4 \le 0$

3.4 22. Find the x- and y- intercepts of: a) $4x^2 - 3y^2 = 16$ and b) $y = \dfrac{2x + 6}{x - 1}$

1.1 23. Write the equation of a circle if (1, 3) and (-5, 7) are end points of a diameter.

5.3 24. Find the domain of the function given by $f(x) = \dfrac{\ln(x-1)}{\sqrt{16 - x^2}}$.

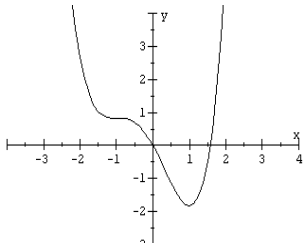
2.4 25. Given the graph below, state whether the graph has x-axis, y-axis, or origin symmetry.



2.4 26. Use tests for symmetry to determine if $x + 2 = 3y^2$ has x-axis, y-axis, or origin symmetry.

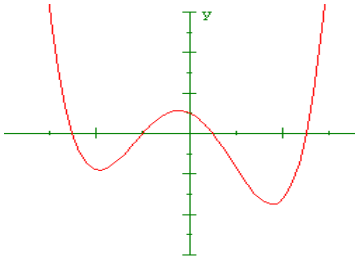
2.2 27. Given $f(x) = -x^2 - 2x + 3$, find $\dfrac{1}{f(2)}$; $f(a+1)$; and $\dfrac{f(x+h) - f(x)}{h}$.

1.2 28. Given the graph of $y = f(x)$, estimate: a) $f(x)$ when $x = 1$; b) x when $f(x) = 1$.

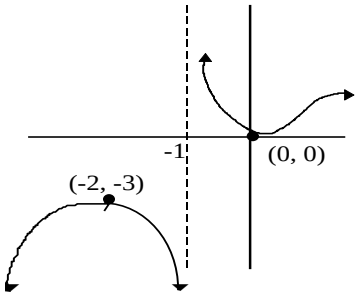


y = f(x)

1.2 29. Is the given graph a function?



1.2 30. Use the graph to find the domain and range of the function:



Domain _____

Range _____

2.2 31. Find the domain of: $f(x) = \dfrac{\sqrt{2x-5}}{x-7}$. Write your answer using interval notation.

2.1 32. Draw the graph of $f(x) = 3x^3 - 8x^2 - 9x + 4$ Determine the intervals on which f is increasing; decreasing.

2.1 33. Draw the graph of $f(x)$: $f(x) = \begin{cases} -2x-5 & x < -2 \\ x^2 & -2 \le x \le 3 \\ 3 & x > 3 \end{cases}$

1.3 34. Suppose that $f(x)$ is a linear function with $f(3) = -7$ and $f(-5) = -2$. Find $f(x)$.

1.4 35. Write the equation of the perpendicular bisector of segment $\overline{AB}$ given $A(3,5)$ and $B(7,6)$.

2.5 36. State the transformations necessary to graph $f(x) = 3(x-2)^2 + 1$ from $f(x) = x^2$, then draw the graph.

3.3 37. $f(x) = 2x^2 - 7x + 3$

a) Complete the square to find the vertex.

b) Find the line of symmetry.

c) Find the range of the function.

d) Does the function have a maximum or minimum?

3.3 38. If a quadratic function has vertex (2, 3) and passes through point (4, 7), find the equation of the function.

2.3 39. If $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 2$, find: a) $(f + g)(x)$

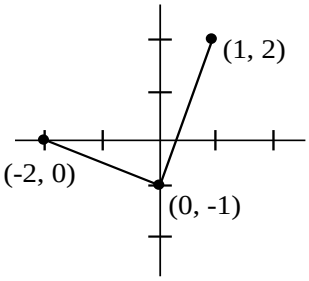
b) (fg) (5)

c) $(f \circ g)$ (5)

d) $(f \circ g)$ (x)

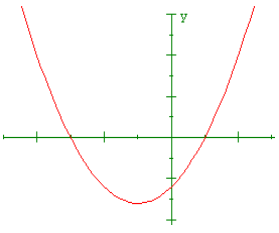
2.5 40. Given the graph of $y = f(x)$ draw the graph of

$y = 2 + 3f(x-1)$.



2.5 41. If $f(x) = x^2$ write the equation of the quadratic if the following geometric transformations are applied: Vertical stretch by a factor of 2, reflect through the x- axis, horizontal shift 3 left and vertical shift 5 down.

5.1 42. Is the given graph one to one?



5.1 43. Find the domain and range for the inverse of f if $f(x)=\sqrt{x-3}$

5.1 44. Find the rule for the inverse of f if $f(x)=\dfrac{2x-3}{x+5}$

5.1 45. Draw the graph of f and its inverse on the same set of axes: $f(x)=x^2+5;\ \ (x\geq 0)$

2.1 46. a) Draw a complete graph of $f(x)=2x^3-7x^2-5x+4.$

b) State coordinates of all local maximum and minimum points.

c) State intervals where graph is increasing; decreasing.

(for parts b and c, round each coordinate/interval endpoint to 2 decimal places.)

2.4 47. State whether the function $f(x)=\dfrac{3x}{x^2+5}$ is odd, even, or neither.

4.3 48. Use synthetic division to divide $4x^3-7x^2+2$ by $x-3$ and state the quotient and the remainder.

4.3 49. Find the remainder if *f*(*x*) is divided by $x+1$: $f(x)=3x^{75}-4x+6$

4.3 50. Find the quotient *Q*(*x*) and remainder *R*(*x*) if $P(x)=x^3+2x^2+5$ is divided by

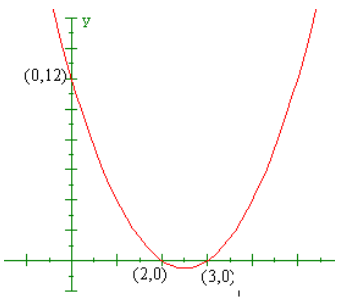
$d(x)=x^2-4.$

4.3 51. Determine if $x-3$ is a factor of $2x^3-13x^2+23x-6$

4.3 52. Given that f $f(x)=x^2+(k-1)x-3k-17$ has $x+2$ as a factor, find *k*.

4.4 53. Write a polynomial of degree 4 with real coefficients and zeroes of -2, 3, and 4 + *i* (DO NOT MULTIPLY OUT).

4.4 54. Write a 2nd degree polynomial f with real coefficients if the graph of $y=f(x)$ is as given:



3.1 55. Simplify: $2(3 + 5i) - 4(3 - 2i)$

3.1 56. Simplify: $\dfrac{3-7\mathrm{i}}{2+5\mathrm{i}}$

4.4 57. Find all zeroes of *f* over the field of complex numbers: $f(x) = x^4 + 6x^3 - 3x^2 + 24x - 28$

4.4 58. Find to the nearest 0.01 all zeroes of f over the field of real numbers:

$f(x) = 3x^3 - 2x^2 - 7x + 3$

4.5 59. Given the rational function: $f(x) = \dfrac{3x}{x^3 + 5x^2 + 4x}$.

a) List all vertical asymptotes.

b) List all horizontal asymptotes.

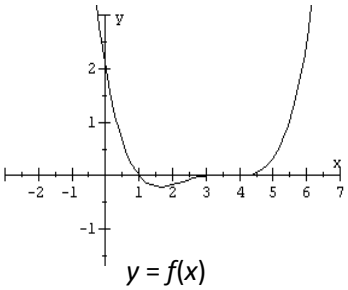
c) Find the domain of *f*.

3.4 60. Let $f(x) = \dfrac{3x-1}{x-2} + \dfrac{4x+5}{x+3}$ and $g(x) = \dfrac{35}{x^2 + x - 6}$. Find all numbers x such that

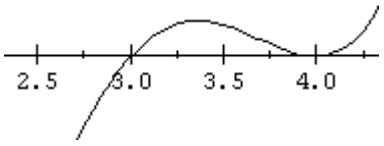
$f(x) = g(x)$.

4.5 61. Draw a complete graph of $y = f(x)$ showing (a) all zeroes, (b) *y*- intercept, (c) vertical asymptotes, (d) horizontal asymptotes, (e) oblique asymptotes. $f(x) = \dfrac{x^3 + 3x^2 - 6x - 8}{x^2 - 3x - 10}$

4.4 62. Given the graph of $y = f(x)$, write a polynomial function *f* of least degree:



y = f(x)



detail of *y = f(x)* on [2.5,4.5]

2.3 63. Let $f(x) = \sqrt{x-5}$ and $g(x) = x^2 + 1$. Find the domain of $(f \circ g)(x)$ and $(g \circ f)(x)$.

3.4 64. Solve algebraically: $\sqrt[3]{2\mathrm{x} - 5} + 1 = 3$

3.4 65. Solve algebraically: $\sqrt{3x-5}+1=\sqrt{x+6}$

2.5 66. Draw the graph of $f(x)=3\sqrt{x-5}-4$ by applying geometric transformations to the graph of $y=\sqrt{x}$.

5.2 67. Sketch a graph of $y=3^{(x+1)}-2$ by applying geometric transformations to the graph of $y=3^x$. State the domain, range and any asymptotes.

5.5 68. Solve: $9^{(x^2)}=3^{3x+2}$ without calculator.

5.5 69. Graph f on the interval [-3, 3]; find all maxima and minima. $f(x)=\dfrac{3.1^{-x}-4.1^x}{4.4^{-x}+5.3^x}$

5.4 70. Evaluate: a) $\log_4(-2)$ b) $\log_3(81)$ c) $2^{\log_2 8}$

5.3 71. Sketch the graph of $f(x)=\log_5(x+1)-2$. State the domain, range and asymptotes.

5.5 72. Solve without calculator: $\log_3(7x+3)=\log_3(x-4)+2$

5.4 73. a) Change $\log_3 X=P$ to exponential form;

b) Change $3^Q=R$ to logarithmic form.

5.4 74. Express in terms of sums and differences of logarithms: $\log\dfrac{ab^2}{\sqrt{2c}}$

5.4 75. Write the expression as one logarithm: $5\log_4 x-1/2\ \log_4(3x-4)-3\log_4(5x+1)$

5.5 76. Solve exactly: $5(4^{2x-1})=21$

5.5 77. Solve graphically: a) $4^{2x+3}=5^{x-2}$ b) $\ln x+\ln(x-2)=5$

5.4 78. Evaluate: $\log_5 17$

5.4 79. Suppose it is given that $\log_3(2)=0.63$ and $\log_3(34)=3.21$. Use these, and appropriate properties of logarithms, to find the values of the following logarithms.

(a) $\log_3(36)$ (b) $\log_3(32)$ (c) $\log_3\left(\frac{1}{17}\right)$ (d) $\log_3\left(\sqrt[3]{68}\right)$

5.3 80. Find the domain of the function: (a) $f(x)=\dfrac{x}{\log x}$ (b) $f(x)=\log_7\left(2-\dfrac{x+5}{x-6}\right)$.

2.1 81. From a rectangular piece of cardboard having dimensions 25 inches by 33 inches, an open box is to be made by removing squares of area x^2 from each corner and turning up the sides. Find the value of *x* so the volume is a) 1400 cubic inches b) maximal.

2.1 82. An aquarium of height 2 feet is to have a volume of 10 ft^3. Let *x* denote the length of the base and *y* the width.

a) Express *y* as a function of *x*.

b) Express the total number *S* of square feet of glass needed as a function of *x* (4 sides & bottom are glass -- no top).

5.6 83. In May 2010, a 1932 painting, by Pablo Picasso, sold at a New York City art auction for $106.5 million to an anonymous buyer. This is a record price for any work of art sold at auction. The painting had belonged to the estate of Sydney and Francis Brody, who bought it for $17,000 in 1952 from a New York art dealer, who had acquired it from Picasso in 1936. (Sources: Associated Press, "Picasso Painting Fetches World Record $106.5M at NYC Auction," by Ula Ilnytzky, May 4, 2010; Online Associated Newspapers, May 6, 2010) Assuming that the value A_0 of the painting has grown exponentially:

a) Find the value of k, and determine the exponential growth function $A(t) = A_0e^{kt}$ assuming A_0 = 17,000 and t is the number of years since 1952.

b) Estimate the value of the painting in 2020.

c) What is the doubling time for the value of the painting?

d) After how long will the value of the painting be $240 million, assuming there is no change in the growth rate?

3.2 84. A rectangular plot of ground having dimensions 30 feet by 36 feet is surrounded by a walk of uniform width. If the area of the walk is 1000 sq. ft., what is its width?

4.6 85. The braking distance *d* (in feet) of a certain car traveling *v* (mi/hr) is given by: $d = v + \dfrac{v^2}{20}$. Determine the velocities that result in braking distance of less than 80 feet.

3.3 86. If a ball is thrown up from the roof of a 100 foot building at an initial velocity of 25 ft/sec:

a) What is its maximum height? $\left(s(t) = -16t^2 + v_0t + s_0\right)$

b) How long does it take to hit the ground? (round all answers to the nearest hundredth)

3.3 87. A farmer wishes to put a fence around a rectangular field and then divide the field into three rectangular plots by placing two fences parallel to one of the sides. If the farmer can afford only 2000 yards of fencing, what dimensions will give the maximum rectangular area?

5.6 88. The amount of bacteria in a certain culture triples every 2 hours. Assuming growth is exponential and there are 500 bacteria to start, how many will be present after 6 hours?

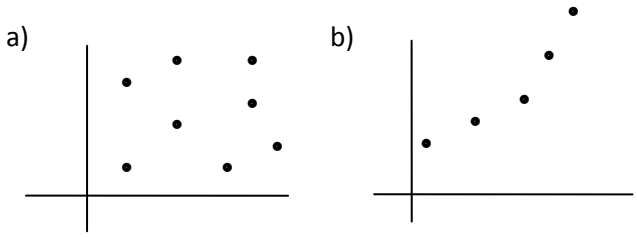
5.6 89. A certain radioactive substance decays according to a formula $q(t) = q_0 e^{-0.0063t}$, where q_0 is the initial amount of the substance and *t* is the time in days. Approximate the half-life of the substance.

1.5 90. My bowling average is 156. So far tonight, I have bowled lines of 140 and 162. What score must I have on the third line to bowl at least my average for the game? A game is 3 lines.

5.2 91. Find the interest rate so that \$2000 grows to \$3000 in 5 years if interest is compounded quarterly.

1.3 92. A small business purchases a piece of equipment for \$25,000. After 10 years, it will have to be replaced. Its value then is expected to be \$2,000. Use a linear equation to find when its value will be \$13,000.

5.6 93. Consider the scatter plot. Determine which (if any), of these functions could be used as a model for the data.



i) Linear

ii) Quadratic

iii) Polynomial but not linear or quadratic.

iv) Exponential

5.6 94. A company introduced a new software product on a trial run in a city. They advertised the product on television and found the following data regarding the percent P of people who bought the product after x ads were run.

Number of Ads, x	Percentage Who Bought, P
0	0.2%
10	0.7%
20	2.7%
30	9.2%
40	27.0%
50	57.6%
60	83.3%
70	94.8%
80	98.5%
90	99.6%

a) Use a graphing calculator to fit a logistic function $P(x) = \frac{a}{1+be^{-kx}}$ to the data.

b) What percent of people bought the product when 55 ads were run? 100 ads?

c) Find the horizontal asymptote for the graph. Interpret the asymptote in terms of the advertising situation.

4.1 95. The time *t* required to empty a tank varies inversely as the rate *r* of pumping. If a pump can empty a tank in 35 minutes at the rate of 200 gal/min, how long will it take to empty the same tank at the rate of 500 gal/min?

3.3 96. An open box is made from a 10-cm by 20-cm piece of aluminum by cutting a square from each corner and folding up the edges. The area of the resulting base is 90 cm^2. What is the length of the sides of the squares?