MATH 121
Real Numbers – Chapter R

1. Compile a list of 12 real numbers, in order, such that all of the following conditions are met:
   a) Each number is contained in the interval $(1.9,\ 2)$.
   b) Six numbers are rational, and six numbers are irrational.
   c) One number is written as a fraction, one as a mixed number, one as a terminating decimal, one as a repeating decimal.
   d) One number is between $1\frac{19}{21}$ and $1\frac{20}{21}$.

2. (a) If one exists, find a real number between .701 and .70101.
   (b) Find a real number between .701 and the number you produced in part (a).
   (c) How many real numbers are there between .701 and .70101?
   (d) Between any two distinct real numbers, how many numbers are there?
   (e) Can you find a real number between $.\overline{9}$ and 1?
   (f) What can you conclude from part (e)?

3. a) Give an example of an irrational number a between the integers 1 and 2.
   b) Now, give an example of another irrational number *b* between *a* and 2.
   c) Now, give an example of another irrational number *c* between *a* and *b*.

   Based on your results from parts (a) through (c) above, complete the following:

   d) Between any two rational numbers I can always find_____

4. FACT: $\sqrt{17}$ is an irrational number.
   a) Using only multiplication (use a calculator but not the "square root" function), estimate $\sqrt{17}$ to the nearest thousandth.
   b) Given that the segment below is 1 unit long, how could you construct a segment that is $\sqrt{17}$ units long? Hint: Consider right triangles.

   _____

5. True or False? If false, provide a counterexample.

   a) Every real number can be expressed as a decimal.
   b) Every real number can be expressed as a terminating decimal or a repeating decimal.
   c) Every real number that can be expressed as a terminating decimal or a repeating decimal is really a fraction of integers in disguise.

6. a) Give an example of an rational number (in fractional form) between $1/3$ and $1/2$.

b) Give an example of another rational number (in fractional form) between $2/3$ and $3/4$.

c) Give an example of another rational number in (in fractional form) between $5/6$ and $6/7$.

d) Based on your results form parts (a) through (c) above, complete the following:
Between any two rational numbers I can always find_____

7. Write a number between 13.252 and 13.2521