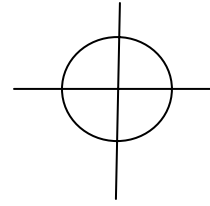


MATH122
Final Exam Review

- 6.2 1. If c represents the length of the hypotenuse, solve the right triangle with $a = 6$ and $B = 71^\circ$.
- 6.1 2. If $\sec \theta = \frac{8}{3}$ and θ is an acute angle, find $\sin \theta$ exactly.
- 6.1 3a. Convert $24^\circ 17' 40''$ to decimal degree notation.
b. Convert 49.254 to degree, minute, second notation. Round to the nearest second.
- 6.1 4. Find the acute angle θ in decimal degrees to the nearest hundredth.
a. $\cos \theta = 0.6437$
b. $\sec \theta = 5$
- 6.1 5. Find the value accurate to the nearest hundredth.
a. $\cos 15.6^\circ$
b. $\cot 47.1^\circ$
- 6.1 6. Fill in the blanks. $\cos 81^\circ = \frac{1}{\underline{\hspace{1cm}}} 81^\circ = \underline{\hspace{1cm}} 9^\circ$
- 6.2 7. You are a tour guide lost in the Rockies and have just come to a gorge. You estimate the angle of elevation of the top of the useful part of a tree that is 25 feet away from you to be 70° . You also estimate that the gorge is 50 feet wide and want to allow an extra 6 feet on each end for safety. Will the useable part of the tree be long enough to bridge the gorge if you cut the tree? (Note: the tree just happens to be standing six feet from the edge of the gorge.)
- 6.2 8. You are estimating the height of a lighthouse surrounded by water. When you first observe it from the shoreline, the angle of elevation seems to be about 16° . After walking inland 200 feet in a direct line with the first observation point, the angle of elevation is about 10° . Find the height of the lighthouse if the ground is level.
- 6.2 9. A ship sails $S 49^\circ E$ for two hours at 34 km per hour. How far east is it of its original position?
- 6.3 10. Find one positive and one negative angle coterminal with 237° .
- 6.3 11. Find exactly the six trig function values for an angle in standard position if the point $(2, -6)$ is on its terminal side.
- 6.3 12. Find $\sin \theta$ where θ is the angle in standard position whose terminal side is the part of $2x - 7y = 0$ in quadrant III.
- 6.3 13. Find the angle θ in the interval $(270^\circ, 360^\circ)$ that has $\cos \theta = 0.4761$.
- 6.3 14. Find the exact value of $\cos 45^\circ$; $\tan 150^\circ$.
- 6.3 15. Evaluate $\sin 96^\circ$; $\csc 237^\circ$.

6.3 16. A plane flies 3.5 hours at 250 miles per hour in a direction of 127° . How far east is it from its starting position?

6.4 17. Mark the point determined by 6 on the unit circle.



6.4 18. Find one positive and one negative angle coterminal with $\frac{3\pi}{7}$.

6.4 19. Find the complement and supplement of $\frac{2\pi}{5}$.

6.4 20. Convert

a. $\frac{3\pi}{4}$ radians to degree measure.

b. 196° to radian measure. Round to the nearest hundredth.

6.4 21. The roller on a treadmill has a diameter of 3 inches. How many revolutions per minute must it make in order for Josh to run at a speed of 10 miles per hour? (1 mile = 5,280 feet)

6.5 22. Find the exact value. a. $\tan \frac{\pi}{6}$ b. $\sec \frac{5\pi}{4}$ c. $\cot 0$

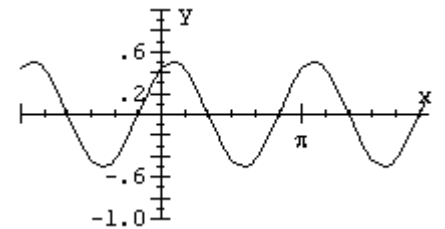
6.5 23. Find the value accurate to the nearest hundredth. a. $\sin 4$ b. $\csc(-1.8)$

6.5 24. Sketch a graph of $y = \sin(-x)$ by hand.

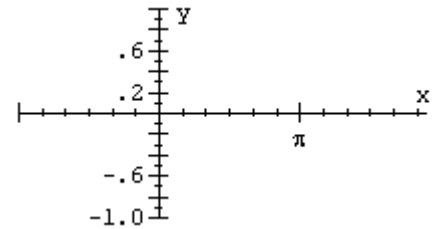
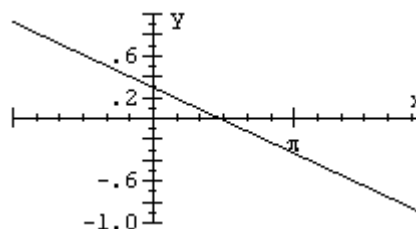
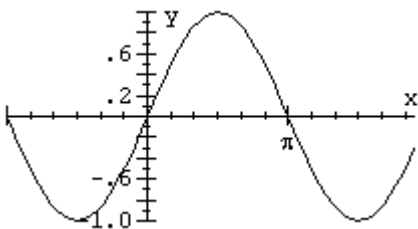
6.6 25. Find the amplitude, period, and phase shift of the graph of $y = 2 \sin\left(3x + \frac{\pi}{2}\right) - 1$.

6.6 26. Graph the function by hand. a. $y = 4 \cos(3x)$ b. $y = \sin\left(x + \frac{\pi}{3}\right)$ c. $y = 2 \sin(3x - \pi)$

6.6 27. Write the equation of the graph in the form $y = A \sin(Bx + C)$.



6.6 28. Graph by adding ordinates.



7.1 29. Write an equivalent expression using only one trig function; evaluate exactly.

$$\sin 61^\circ \cos 74^\circ + \cos 61^\circ \sin 74^\circ$$

- 7.1 30. Rationalize the denominator; simplify if possible. a. $\sqrt{\frac{2 \sin x}{\cos x}}$ b. $\sqrt{\frac{\sin^2 x}{1 - \cos x}}$
- 7.1 31. Use the substitution $x = 5 \sin \theta$ to rewrite $\sqrt{25 - x^2}$ without using a radical.
- 7.1 32. Find $\cos 105^\circ$ exactly. (Hint: use a sum or difference identity)
- 7.1 33. Given that $\sin A = \frac{2}{5}$; $\frac{\pi}{2} < A < \pi$ and $\tan B = \frac{2}{3}$; $\pi < B < \frac{3\pi}{2}$, find $\cos(A - B)$ exactly.
- 7.2 34. If $\cos \theta = \frac{1}{3}$; $\frac{3\pi}{2} < \theta < 2\pi$, find a. $\sin 2\theta$ b. $\cos \frac{\theta}{2}$
- 7.2 35. Simplify $\frac{\cos^2 \theta}{1 + \sin \theta}$.
- 7.3 36. Verify the trig identity. $\sec^4 x - \tan^4 x = 1 + 2 \tan^2 x$
- 7.4 37. Evaluate $\sin^{-1}\left(\frac{-1}{2}\right)$ exactly in radians.
- 7.4 38. Evaluate exactly. a. $\sin^{-1}\left(\sin \frac{3\pi}{2}\right)$ b. $\cos\left(\sin^{-1} \frac{2}{3}\right)$
- 7.4 39. Simplify so that the answer contains no trig functions.
 a. $\tan\left(\cos^{-1} \frac{b}{2}\right)$ b. $\sin\left(\tan^{-1} \frac{a}{7} + \cos^{-1} \frac{2}{3}\right)$
- 7.4 40. Find all real solutions exactly in radians.
 a. $\sqrt{3} \sec x + 2 = 0$ b. $2 \sin^2 x - \sin x = 1$ c. $\sin^2 x + \cos x - 1 = 0$
- 7.4 41. Find all real solutions. $2 \cos x - x^2 = 0$
- 8.1 42. Solve the triangle in which a. $a = 4$, $B = 27^\circ$, $C = 102^\circ$ b. $a = 10$, $B = 27^\circ$, $b = 6$
- 8.1 43. A pilot is about to land on the 8500 foot runway in Los Angeles. The angles of depression from the plane to the ends of the runway are 16° and 17.2° . Find the distance the plane must fly before it touches down.
- 8.2 44. Solve the triangle in which $a = 20$, $b = 27$, $c = 16$.
- 8.2 45. The distance between two planes is calculated by using LORAN to find the position of each plane relative to the tracking station. If one plane is 30 miles from the station, a second is 27 miles, and the angle between the two distances is 37° , find how far apart the planes are.

- 8.4 46. Convert the point $\left(3, \frac{4\pi}{3}\right)$ from polar to rectangular coordinates.
- 8.4 47. Convert the point $(2, -2)$ from rectangular to polar coordinates.
- 8.5 48. Graph the point $\left(-3, \frac{5\pi}{6}\right)$ in a polar coordinate system.
- 8.5 49. Convert $r = 6 \sin \theta$ to a rectangular equation.
- 8.5 50. Convert $x^2 + (y - 5)^2 = 25$ to a polar equation.
- 8.5 51. Graph $r = 5 \sin 3\theta$.
- 8.5 52. Find the magnitude of $\mathbf{u} + \mathbf{v}$ and the angle between that sum and \mathbf{u} where $|\mathbf{u}| = 25, |\mathbf{v}| = 19$, and the angle between \mathbf{u} and \mathbf{v} is 75° .
- 8.5 53. A plane is to make a flight to New York, which is a direction of 108° from Detroit. The plane has an airspeed of 175 miles per hour in still air, and there is a tail wind of 40 miles per hour blowing from a direction of 308° . Find the heading the plane must use in order for it to reach its destination.
- 8.6 54. Write the vector \overline{AB} in component form. A and B are the points $(3, -7)$ and $(-1, 4)$ respectively.
- 8.6 55. Given $\mathbf{u} = \langle 1, -2 \rangle$, $\mathbf{v} = \langle 4, 3 \rangle$, find $2\mathbf{u} - 3\mathbf{v}$.
- 8.6 56. Find a unit vector in the same direction as $3\mathbf{i} - 4\mathbf{j}$.
- 8.6 57. Find the direction angle of the vector $\langle -3, 4 \rangle$.
- 8.6 58. Find the angle between $\langle -3, 4 \rangle$ and $\langle 7, 12 \rangle$.
- 8.6 59. A plane flies at a heading of 12° a distance of 100 miles, lands, then takes off again at a heading of 140° , travels 150 miles in that direction, and lands a second time. What heading and distance would a direct flight use?
- 9.1 60. Solve the system.
$$\begin{aligned} 4x + 3y &= -3 \\ 3x + 2y &= -1 \end{aligned}$$
- 9.1 61. A chemist needs 100 ml of a 27% solution of sulfuric acid, but only has 10% and 40% solutions available. Find how much of each should be used.
- 9.2 62. Solve the system using any method.
$$\begin{aligned} 2x + y + 3z &= 12 \\ 5x + 2y + z &= 5 \\ -3x + 3y + 2z &= -1 \end{aligned}$$

- 9.2 63. Michigan State University and the University of Michigan scored a total of 40 points during a 1994 game. There were 11 scoring plays: touchdowns, extra point kicks, and field goals at 6, 1, and 3 points respectively. There were twice as many extra points as field goals. Find the number of each scoring play for the game.

$$0.6x + 1.2y + 4z = -4.82$$

- 9.3 64. Solve the system using augmented matrices on your calculator. $2x + 5y + 5z = 23$

$$3.1x - 1.9y - 6z = 16.49$$

- 9.3 65. Solve the system.
- | | | |
|----|------------------|------------------|
| | $3x + y - z = 2$ | $3x + y - z = 2$ |
| a. | $x - y + 2z = 1$ | $x - y + 2z = 1$ |
| b. | $x - y + 2z = 1$ | $x - y + 2z = 1$ |
| | $7x + y = 5$ | $7x + y = 2$ |

- 11.1 66. List the first four terms of the sequence $\left\{ \frac{n}{2n+1} \right\}$.

- 11.1 67. Find a formula for the n th term of the sequence $1, \frac{3}{5}, \frac{4}{10}, \frac{5}{17}, \frac{6}{26}, \dots$

- 11.1 68a. Find the fourth partial sum of the sequence $3, 12, 27, 48, 75, \dots$ b. Evaluate the sum $\sum_{k=1}^4 3k^2$.

- 11.1 69. Write sigma notation for the series associated with the sequence: $\frac{3}{2}, \frac{5}{3}, \frac{7}{4}, \frac{9}{5}, \frac{11}{6}, \dots$

- 11.1 70. Find the first five terms of the sequence $\{a_n\}$ where $a_1 = 3$, $a_{n+1} = 2a_n + 1$ for $n \geq 1$.

- 11.2 71. Find the 31st term of the sequence $-8, -5, -2, 1, 4, \dots$

- 11.2 72. Find a_{23} for the arithmetic sequence $\{a_n\}$ which has $a_7 = 27$ and $a_{40} = 159$.

- 11.2 73. Evaluate the sum $\sum_{k=1}^{23} \frac{2k+1}{3}$.

- 11.2 74. Each row in an auditorium seats 2 more people than the preceding row. There are 25 seats in the first row and 48 rows. Find the seating capacity of the auditorium.

- 11.3 75. Find the 10th term of the geometric sequence $12, 18, 27, \frac{81}{2}, \dots$

- 11.3 76. Find a formula for the n th (general) term of the sequence $24, -12, 6, -3, \dots$

- 11.3 77. Find the sum. $\sum_{k=1}^{12} 48 \left(\frac{1}{2} \right)^k$

- 11.3 78. Find the sum. $\sum_{k=1}^{\infty} 48 \left(\frac{1}{2}\right)^k$
- 11.3 79. Write the corresponding fraction for the decimal 4.1525252...
- 11.3 80. You have a job offer with a salary of \$30,000 for the first year. If you accept the job and get a 2.5% raise every year for the next 30 years, what are your total earnings for the entire 30 years?
- 11.5 81. Evaluate. a. ${}_{11}P_3$ b. $\frac{10!}{6!}$
- 11.5 82. How many different ways can twelve horses come in first, second, and third in a race in which there are no ties?
- 11.6 83. Evaluate. a. $\binom{6}{4}$ b. ${}_8C_3$
- 11.6 84. There are 50 numbers in the Michigan state lottery. How many ways are there to select six of them?
- 11.7 85. Expand and simplify a. $(2x-3)^5$ using the binomial theorem b. $\left(\frac{1}{x}+2\right)^4$ using Pascal's triangle.
- 11.7 86. Find the tenth term in the binomial expansion of $\left(\frac{2}{x^2}-\frac{x}{2}\right)^{17}$.
- 11.7 87. The probability that a player hitting .250 will make a hit 3 times in the next eight at bats is the term in which .25 has an exponent of 3 in the binomial expansion of $(.25+.75)^8$. Find that probability.
- 10.1 88. Find the vertex, focus, and directrix of the parabola $x^2+6x+4y+1=0$. Draw the graph.
- 10.1 89. A parabolic reflector for a telescope is six inches in diameter. It is ground so that its vertex is 0.15 inch below the rim (on the concave side). a. Insert an xy -coordinate system with vertex at the origin and find the equation of the parabola. b. How far is the focus from the vertex?
- 10.2 90. Find the center and radius of the circle with equation $(x+1)^2+(y-1)^2=4$.
- 10.2 91. Find the vertices and foci of the ellipse with equation $4x^2+9y^2=36$.
- 10.2 92. Write the equation of an ellipse centered at the origin containing the point $(4,0)$ and with one focus at $(0,3)$.

- 10.3 93. Find the center, vertices, foci, and asymptotes of $25x^2 - 4y^2 + 200x + 8y + 296 = 0$. Draw the graph.
- 10.7 94. Graph the curve $x = 3 \cos t$, $y = 2 \sin t$. Find an equivalent rectangular equation.
- 10.7 95. A ball is thrown with an initial speed of 92 feet per second from a height of 7 feet at an angle of 50° to the horizontal. You may assume that the ground is level. a. Find the height of the ball after 2 seconds. b. Determine the horizontal distance the ball travels.