MATH 253 – Formulas

CONVERSION	FORMULAS
$(r, \theta, z) \to (x, y, z)$ $(x, y, z) \to (r, \theta, z)$	$x = r\cos\theta,\ y = r\sin\theta,\ z = z$ $r = \sqrt{x^2 + y^2}$, $\tan\theta = y/x,\ z = z$
$(\rho, \theta, \varphi) \to (r, \theta, z)$ $(r, \theta, z) \to (\rho, \theta, \varphi)$	$r = \rho\sin\varphi,\ \theta = \theta,\ z = \rho\cos\varphi$ $\rho = \sqrt{r^2 + z^2}$, $\theta = \theta,\ \tan\varphi = r/z$
$(\rho, \theta, \varphi) \to (x, y, z)$ $(x, y, z) \to (\rho, \theta, \varphi)$	$x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta,\ z = \rho\cos\varphi$ $\rho = \sqrt{x^2 + y^2 + z^2}$, $\tan\theta = y/x,\ \cos\varphi = z/\sqrt{x^2 + y^2 + z^2}$

$\iiint dV = \iiint \rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi$

Dot Product: If $\vec{u} = \langle a_1, a_2, a_3 \rangle$, $\vec{v} = \langle b_1, b_2, b_3 \rangle$, then $\vec{u} \bullet \vec{v} = |\vec{u}||\vec{v}|\cos\theta = a_1 b_1 + a_2 b_2 + a_3 b_3$

The unit vector in the direction of $\vec{v}$ is: $\vec{u} = \dfrac{\vec{v}}{|\vec{v}|}$

Cross Product: $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2)\vec{i} - (a_1 b_3 - a_3 b_1)\vec{j} + \left(a_1 b_2 - a_2 b_1\right)\vec{k}$

Parametric Equations of a Line: $x = at + x_0, y = bt + y_0, z = ct + z_0$

Equation of a Plane: $Ax + By + Cz = d$

Linearization: $L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Total Differential: $df = f_x(x_0, y_0, z_0)\,dx + f_y(x_0, y_0, z_0)\,dy + f_z(x_0, y_0, z_0)\,dz$

Chain Rule for Functions of Two Independent Variables, $w = f(x, y)$: $\dfrac{dw}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$

Tangent Line to a Level Curve: $f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) = 0$

Tangent Plane (explicit form): $z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Tangent Plane (implicit form): $F_x(P_0)(x - x_0) + F_y(P_0)(y - y_0) + F_z(P_0)(z - z_0) = 0$

or: $F_x(P_0)\,x + F_y(P_0)\,y + F_z(P_0)\,z = F_x(P_0)\,x_0 + F_y(P_0)\,y_0 + F_z(P_0)\,z_0$

Gradient Vector: $\vec{\nabla} f = \langle f_x, f_y, f_z \rangle = \dfrac{\partial f}{\partial x}\vec{i} + \dfrac{\partial f}{\partial y}\vec{j} + \dfrac{\partial f}{\partial z}\vec{k}$

Directional Derivative: $D_{\vec{u}} f\left(x, y, z\right) = \vec{\nabla} f(x, y, z) \bullet \vec{u}$, where $\vec{u}$ is a unit vector.

Implicit Differentiation: $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y}$ or $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z}$, $\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z}$

Second Derivative Test: Let $D = f_{xx}(a,b)\cdot f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$

(a) If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum.
(b) If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum.
(c) If $D < 0$, then $f(a,b)$ is not a local maximum or minimum.
(d) If $D = 0$, then the test gives no information.

Length of a Parametric Curve: $L = \int_C \left|\vec{r}'(t)\right| dt = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$

Surface Area of a Parametric Surface: $A(s) = \iint_D \left|\vec{r}_u \times \vec{r}_v\right| dA$, or $A(s) = \iint_D \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$

Center of Mass:

$$\bar{x} = \frac{1}{m}\iiint x\rho(x,y,z)dv, \quad \vec{y} = \frac{1}{m}\iiint y\rho(x,y,z)dv, \quad \vec{z} = \frac{1}{m}\iiint z\rho(x,y,z)dv,$$

where $m = \iiint \rho(x,y,z)\, dv$ is the mass.

Conservative Vector Fields: $\vec{F} = P(x,y)\vec{i} + Q(x,y)\vec{j}$ is conservative iff $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

$\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$ is conservative iff $curl\left(\vec{F}\right) = \vec{\nabla}\times\vec{F} = \vec{0}$

Line Integral of f over a Curve **C:** $\int_C f(x,y,z)\, ds = \int_a^b f(x(t),y(t),z(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$

Line Integral of **F** over a Curve **C:** $\int_C \vec{F}\bullet d\vec{r} = \int_C \vec{F}\bullet\vec{r}'(t)\, dt = \int_C Pdx + Qdy + Rdz$

The Fundamental Theorem for Line Integral: $\int_C \vec{\nabla} f \bullet d\vec{r} = [f]_{\vec{r}(a)}^{\vec{r}(b)} = f\left(\vec{r}(b)\right) - f\left(\vec{r}(a)\right)$

Curl and Divergence of **F:** $\text{curl}\left(\vec{\text{F}}\right) = \vec{\nabla}\times\vec{F}$, $\text{div}\left(\vec{F}\right) = \vec{\nabla}\bullet\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

Green's Theorem: $\oint_C (Pdx + Qdy) = \int\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

Surface Integral of f over a Surface **S:** $\iint_S f(x,y,z)\, dS = \iint_D f\left(\vec{u}(u,v)\right)\left|\vec{r}_u \times \vec{r}_v\right| dA$

Flux (Surface Integral of **F** over **S**): $\iint_S \vec{F}\bullet d\vec{S} = \iint_S \vec{F}\bullet\vec{n}\, dS = \iint_D \vec{F}\bullet\left(\vec{r}_u \times \vec{r}_v\right) dA = \iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA$

Stoke's Theorem: $\int_C \vec{F}\bullet d\vec{r} = \iint_S \text{curl}\,\vec{F}\bullet d\vec{S}$

Divergence Theorem: $\iint_S \vec{F}\bullet d\vec{S} = \iiint_E \text{div}\,\vec{F}\, dV$